Supposition: A Problem for Bilateralism

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Abstract

In bilateral logic formulas are signed by + and −, indicating the speech acts assertion and denial. I argue that making an assumption is also a speech act. Speech acts cannot be embedded within other speech acts. Hence we cannot make sense of the notion of making an assumption in bilateral logic. Attempts to solve this problem are considered and rejected.

Keywords. Assertion, denial, negation, supposition, assumption, speech acts

1 Introduction

According to bilateralist inferentialist semantics for the logical constants, their meanings are determined, not merely by rules of inference specifying their use in deductive arguments, but by rules specifying their use in deductive arguments that appeal to two primitive speech acts of assertion and denial. It is part of a wider position in the theory of meaning, proposed by Price, which, quite generally, 'takes the fundamental notion for a recursive theory of sense to be not assertion conditions alone, but these in conjunction with rejection, or *denial* conditions' (Price, 1983, 162). As Rumfitt puts it, 'mastering the sense of an atomic sentence A will involve learning methods whose deployment might entitle one either to affirm it or to reject it' (Rumfitt, 2000, 797). Accordingly, rules of inference in bilateral logic do not merely specify which conclusions follow from which premises, but they do so in a way that construes premises and conclusions as assertions or denials.[1]

The most prominent system of bilateral logic has been proposed by Rumfitt (2000), building on work by Smiley (1996). Humberstone (2000) proposed a similar system around the same time as Rumfitt. Their formalism has been

[1]I would like to thank Dorothy Edgington, Keith Hossack, Jessica Leech, Eliot Michelson, Jonathan Nassim and Greg Restall for discussion of and comments on this paper and the issues explored therein. It was presented at the Institute of Philosophy of the University of London, where I received helpful comments from Corine Besson, Michael Potter and Bernhard Weiss. It received its final touches after its presentation and discussion at Sara's Ayhan's conference 'Bilateralism and Proof Theoretic Semantics'. A referee for the *Bulletin* made encouraging comments. Preparing the paper for the present volume made me realise how much it owes to comments by Mark Textor: to him, most thanks belong.

taken up by various writers with an interest in inferentialism or proof-theoretic semantics, such as Restall (2005) and Francez (2014).[2]

In this paper I shall point out a fundamental problem for the framework of bilateral logic. In bilateral logic, all formulas are supposed to be asserted or denied. Logical inference involves making and discharging assumptions, as witnessed also by the rules of bilateral logic. Assertion and denial are speech acts. Making an assumption is also a speech act. Hence bilateral logic demands that assertions and denials may be assumed and discharged. But this cannot be done, as speech acts cannot be iterated. Bilateral logic as it stands is thus incoherent.[3]

The final section considers two attempts to solve this problem by incorporating speech acts for supposition within bilateral logic or interpreting deductions in bilateral logic as conditional assertions and denials. I conclude that neither approach is successful.

2 Bilateral Logic

The rules of bilateral logic are applied to asserted or denied formulas. It builds on the claim that there is 'a readily comprehensible variety of actual deductive practice in which the components of arguments express the assignation of affirmative or negative force to propositional contents' (Rumfitt, 2000, 798). Smiley and Rumfitt motivate this by examples of how questions and answers may figure in arguments.

Frege suggested that we can represent the content of a sentence, which we may also call a *proposition* or a *thought*, by a 'propositional question'[4], a question that asks for the answer 'Yes' or 'No'.[5] An assertion can then be effected by answering 'Yes' to such a question. This is as far as Frege went, who did not afford the answers 'Yes' and 'No' the same status, but preferred to keep only 'Yes' as primitive and to treat 'No' as analysed in terms 'Yes' and sentential negation. With some justice bilateralists observe that *prima facie* the answers 'Yes' and 'No' are on a par. Bilateralists hold that, just as an assertion may be effected by the answer 'Yes', a denial may be effected by answering 'No'. According to Smiley, 'a mechanism for rejection is there for anyone who wishes to use it, in the shape of an answer to a yes-or-no question. Questioner and answerer are usually different people, but if one puts the question to oneself, one comes up with the forms "P? Yes" and "P? No". I suggest that "...? Yes?" is a very passable realization of Frege's assertion-sign, the "judgment stroke"

[2]Rumfitt's system was devised with an eye on a formalisation of classical logic that respects Dummettian considerations on harmony. For a different and striking account of a natural proof system for classical logic see Restall (2021).

[3]It has been observed before that treating assumptions like assertions or denials may pose a problem for bilateralist logic, e.g. by Incurvati and Smith (2012, 230), Hjortland (2014, 464, footnote 23) and myself ((2017), (2019, 221)), but as far as I am aware the present paper contains the first sustained discussion of the issue. Although I hope there to be some agreement, the other authors' remarks are too brief for it to be possible to assess whether they would accept the analysis of the source and the precise nature of the problem put forward here. The present paper keeps a promise to expound the details of my objection.

[4]This is Geach's translation of Frege's *Satzfrage* (Frege, 1960, 117).

[5]Frege's view is slightly more nuanced, as he also acknowledges that the propositional question contains more than just the thought, namely the request that the question be answered. This nuance is of no consequence for present purposes. See (Frege, 1918a, 62) and (Frege, 1918b, 145).

in his turnstile notation, and that "...? No" is an equally passable realization of a rejection-sign' (Smiley, 1996, 1). Notice that there are not two things, answering a propositional question with 'Yes' or 'No' and asserting or denying the corresponding declarative sentence: answering 'Yes' to a propositional question just is to assert the thought expressed; answering 'No' just is to deny it.

Rumfitt adapts an example of Smiley's, itself inspired by one of Frege's, to illustrate how propositional questions and their answers, and accordingly assertions and denials, may be used in deductive arguments:[6]

> If the accused was in Berlin at the time of the murder, could he have committed it? No.
> Was the accused in Berlin at the time of the murder? Yes.
> So: Could he have committed the murder? No.

Smiley's example, where $*$ indicates rejection and no star assertion, is:

> If the accused was not in Berlin at the time of the murder, he did not commit the murder.
> $*$The accused was in Berlin at the time of the murder.
> So: $*$ The accused committed the murder.

Transposing it into question and answer format, the result is:

> If the accused was not in Berlin, he did not commit the murder? Yes.
> Was the accused in Berlin at the time of the murder? No.
> So: Did the accused commit the murder? No.

Humberstone observes that Smiley's notation is confusing and does not capture the supposedly equal status of assertion and denial (Humberstone, 2000, 345). It is preferable to introduce two symbols, one for assertion and one for denial. Humberstone and Rumfitt use + and −: 'Where A is a declarative sentence (or formula), let us introduce the signed sentences (or formulae) $+A$ and $-A$ to abbreviate Smiley's amalgams of questions with answers ⌜Is it the case that A? Yes⌝ and ⌜Is it the case that A? No⌝.' (Rumfitt, 2000, 800) Result:

> $+$ If the accused was not in Berlin, he did not commit the murder.
> $-$ The accused was in Berlin at the time of the murder.
> So: $-$ The accused committed the murder.

Thus, bilateralists argue, assertions and denials can be premises and conclusions in deductive arguments.

To specify the meanings of the logical constants in a bilateral inferential semantics, rules of inference must be formulated that specify the conditions

[6] It is questionable whether Rumfitt's is a good example to motivate the bilateral cause. Weiss (2018, 98) observes that if 'No' in the first premise is taken to reject the entire conditional, and that conditional is material, as Smiley and Rumfitt agree it is, then, by bilateral logic, the first premise already entails the conclusion and the second premise is superfluous. For the example to work as one that illustrates a two premise argument with a conclusion, the first premise must be understood as an assertion of the conditional 'If the accused was in Berlin at the time of the murder, he could not have committed the crime', i.e. as the answer 'Yes' to the corresponding propositional question.

under which formulas with the constants as main operators may be asserted and denied.

Rumfitt and Humberstone call the premises and conclusions of the rules of their bilateral logics *signed formulas*, i.e. signed by + and − representing the speech acts of assertion and denial. Lower case Greek letters range over signed formulas. α^* designates the *conjugate* of α, the result of reversing its sign from + to − and conversely. For each connective **c**, there are assertive rules specifying the grounds for and consequences of asserting a formula with **c** as main operator and rejective rules specifying the grounds for and consequences of denying such a formula (Rumfitt, 2000, 800ff). For purposes of illustration it suffices to give only some rules of some of the connectives of Rumfitt's system. The system below is, however, complete in the sense that the missing assertive and rejective rules for $\neg$ and $\supset$ as well as those for the other logical constants, defined as usual in terms of $\neg$ and $\supset$, are derivable:[7]

$$+\!\supset\! I: \dfrac{\begin{array}{c}\overline{+A}^{\;i}\\ \Pi\\ +B\end{array}}{+A\supset B}\;i \qquad\qquad +\!\supset\! E: \dfrac{+A\supset B \qquad +A}{+B}$$

$$-\neg I: \dfrac{+A}{-\neg A} \qquad\qquad -\neg E: \dfrac{-\neg A}{+A}$$

$$\text{Reductio:}\quad \dfrac{\begin{array}{c}\overline{\alpha}^{\;i}\\ \Pi\\ \bot\end{array}}{\alpha*}\;i \qquad\qquad \text{Non-Contradiction:}\quad \dfrac{\alpha \qquad \alpha*}{\bot}$$

Reductio and Non-Contradiction are bilateral versions of common principles, but here they have the character of structural rules governing the framework in which deductions are carried out rather than that of operational rules for logical constants. They codify relations between assertions and denials.

3 Force and Content

Frege distinguishes the *content* of a sentence from the *force* with which it is put forward. Following the widely accepted treatment of this distinction by Hare, Searle and others, the same content can be asserted to be true, it can be asked whether it is true, it can be commanded that it be made true, it can be wished that it were true, etc..[8] Asserting, asking, commanding, wishing are activities speakers engage in: they are speech acts. Different such acts can have

[7] This claim assumes that Rumfitt's requirement that the rule of Non-Contradiction be restricted to atomic premises is not imposed. This restriction is not relevant to what is at issue in the present paper. Kürbis's normalisation proof for Rumfitt's system (2021a) appeals to the unrestricted version, which may speak against imposing it. The proof contains a deplorable oversight, noted with a sketch of a correction in (Kürbis, 2021b)

[8] See, e.g., Hare (1952, Sec 2.1), Searle (1969, 22f, 29ff), Stenius (1967, 1f). Frege's view is once more more nuanced than the received view, as was pointed out to me by Mark Textor. In 'On Sense and Reference', Frege expresses the view that imperatives and optatives do not express thoughts: 'A subordinate clause with "that" after "command," "ask," "forbid," would appear in direct speech as an imperative. Such a clause has no referent but only a sense. A command, a request, are indeed not thoughts, yet they stand on the same level as thoughts. Hence in subordinate clauses depending

the same content. A speech act, being an activity, is not a proposition, and so it is not the kind of thing that can be used as a component in constructing larger propositions by sentential operators. Actions cannot be embedded into contexts that require propositions.[9]

A typical account of why the speech act of assertion cannot form part of propositions is found in Reichenbach's *Elements of Symbolic Logic*:

> Assertion is used in three different meanings: it denotes, first, the act of asserting; second, the result of this act, i.e., an expression of the form '$\vdash p$'; third, a statement which is asserted, i.e. a statement 'p' occurring within an expression '$\vdash p$'. It should be noticed that it is not possible to define the verb 'assert' in terms of the assertion sign. One might suppose that such a definition could be constructed by regarding the sentence '"p" is asserted' as having the same meaning as the expression '$\vdash p$'. But the coordination is not possible because '$\vdash p$' is not a sentence. (Reichenbach, 1966, 346)

Reichenbach then refers to his earlier analysis of the assertion sign as an example of a pragmatic sign. 'Expressions including a pragmatic sign are not propositions. They are not true or false, as is shown by the fact that they cannot be negated. [...] Since assertive expressions are not propositions, they cannot be combined by propositional operations.' (Reichenbach, 1966, 337) This position is widely accepted, in particular by bilateralists.[10]

Speech acts can be described or reported by sentences in the third person[11] such as 'He asked whether p' and 'She asserted that q'. This differs from the performance of the speech act. If I report that she asserted that q, no such speech act with content expressed by 'q' need have been performed: my report may be mistaken. By contrast, if she asserts that q, a speech act with content expressed by 'q' has been performed, no matter whether q is true or false. Sentences describing or reporting speech acts are true or false. Speech acts are performed or not.

In bilateral logic, A represents the content of a speech act, + and − the forces assertion and denial. It makes no sense to put an action into the antecedent of a conditional, for instance: hence the sequence of symbols $(+A) \supset B$ is meaningless. It is crucial that that which is represented by + and − in bilateral logic cannot be embedded:

> It would be a confusion to construe the sign of rejection "−" as a notational variant for the negation operator "¬". Whether in a

upon "command," "ask," etc., words have their indirect referents. The referent of such a clause is therefore not a truth value but a command, a request, and so forth.' (Frege, 1892, 38f) (Black's translation (Frege, 1960, 68).) This is a curious passage and of great interest, but I set it aside. The received view surely has much to be said for it, and Frege's point is orthogonal to present issues in as far as imperatives and optatives are adduced only for heuristic purposes and the focus of this paper lies elsewhere. I set this nuance aside, too.

[9] I set aside the question whether there are mental acts corresponding to speech acts that do not involve linguistic items. The discussion of Frege below mentions judgements, but for present purposes these can be assimilated to assertions.

[10] Of course, now that the option has been mentioned, it may only be a matter of time before someone appears who rejects it.

[11] Sentences in the first person, such as 'I assert that q', by contrast, may achieve both, describe the utterer as performing a speech act and performing it, as pointed out to me by Mark Textor.

formal or a natural language, a sign of negation is a freely iterating sentence-forming operator on sentences: A, ⌜$\neg A$⌝, ⌜$\neg\neg A$⌝, etc. are all well-formed formulae. The sign of rejection, by contrast, was explained as the formal correlate of the operation of forming an interrogative sentence from a declarative sentence and appending the answer "No", and this operation cannot be iterated. "Is it the case that two is not a prime number? No" makes perfectly good sense, but "Is it the case that is it the case that two is a prime number? No? No" is gibberish. The sign "−", then, does not contribute to propositional content, but indicates the force with which that content is promulgated. Just as one asserts the entire content expressed by A by inscribing ⌜$+\,A$⌝, so one expressly rejects that same content by inscribing ⌜$-\,A$⌝. The symbol "+", in a word, is a Fregean assertion sign or Urtheilsstrich; and the symbol "−" is a cognate rejection sign or Verneinungsstrich. (Rumfitt, 2000, 802f)

If expressions such as $(+\,A) \supset B$ or $-\,-\,A$ were legitimate, − and + would be mere notational variants of negation and the truth operator, expressing the trivial truth function mapping True to True and False to False, rather than indicators of the speech acts assertion and denial.

Bilateralists accept what Geach calls the *The Frege Point*: 'A thought may have just the same content whether you assent to its truth or not; a proposition may occur in discourse now asserted, now unasserted, and yet be recognizably the same proposition.' (Geach, 1972, 254f) According to Geach, a phrase cannot carry the assertoric force of an utterance or inscription if a sentence containing that phrase can be embedded into larger sentences, in particular if it can form the antecedent of a conditional: in such a context, the sentence is not asserted, hence the phrase that supposedly carried assertoric force cannot, after all, have done so (Geach, 1972, 262f). The Frege Point provides a test for whether an expression carries the force of a speech act: if a sentence containing the expression can be embedded into a larger sentence so that the speech act is not performed by an utterance of the latter, then the expression cannot carry the force of the speech act.[12]

Geach went so far as to conclude that in ordinary language 'there is no naturally used sign of assertion [...]. That is why Frege had to devise a special sign.' (Geach, 1972, 262f) Bilateralists disagree with Geach's verdict: 'Yes' is such a sign. To argue this point, Rumfitt turns the test provided by the Frege Point into one for signs for speech acts: if a sentence containing a certain expression, or just some individual expression, cannot be embedded, this indicates that the sentence contains, or that the expression is, a sign of a speech act.[13] Other

[12]Geach observes that expressions such as 'the fact that' carry assertoric force even when occurring in embedded sentences. Geach analyses 'Jim is aware of the fact that his wife is unfaithful' as a 'double barrelled assertion' 'equivalent to the pair of assertions "Jim is convinced that his wife is unfaithful" and "Jim's wife is unfaithful".' (Geach, 1972, 259) The occurrence of the phrase 'the fact that' is not, however, a sign carrying the assertoric force of the sentence as a whole, but only of the clause following 'that'. In asserting 'If Jim is aware of the fact that his wife is unfaithful, then he is not showing it', an example I owe to Mark Textor, I do not assert that Jim is aware of the fact that his wife is unfaithful, but only that his wife is unfaithful. Standing alone the phrase 'the fact that' cannot be used to indicate the assertoric force of a sentence, as it forms a noun phrase from a sentence, not a sentence. 'The fact that p obtains' is again not the sign of assertoric force, as in asserting 'If the fact that p obtains, then q', I am not asserting that the fact that p obtains.

[13]This may need qualification, if there are expressions that prevent a sentence containing them

expressions that indicate speech acts are those for greetings, such as 'Hullo', or for valedictions, such as 'Adieu', or for the expression of gratitude, such as 'Thank you': none of these can be embedded.

'It is assertible that' or 'It is deniable that' are not correct renderings of the bilateralist's + and −: sentences beginning with them can be embedded in larger sentences, such as 'If it is assertible that p, then p', 'If it is deniable that p, then $\neg p$', 'It is deniable that it is assertible that p' or 'It is assertible that it is deniable that p'. 'It is assertible that' and 'It is deniable that' are sentential operators. The + and − of bilateral logic are signs that convey the forces of speech acts, in the same category as Frege's judgement stroke.

4 Supposition

One reason why Frege held that the distinction between sense and force is necessary is that it is possible to assume a proposition without asserting it, or, as he put it, without judging it to be true: 'This separation of the judgement from that which is judged appears to be unavoidable, as otherwise a mere assumption, the positing of a case without judging whether it arises, could not be expressed.' (Frege, 1891, 21f) Consequently, Frege explains, he introduces the judgement stroke, a vertical line, to indicate that a proposition is judged or asserted to be true. It is to be put to the left of 'the horizontal', in his early work called 'the content stroke' (Frege, 1879, §2). In a footnote, Frege continues: 'The judgement stroke cannot be used in the formation of a functional expression, because in combination with other symbols it does not serve to designate an object. "$\vdash 2 + 3 = 5$" does not designate anything, it asserts something.' (Frege, 1891, 22) In another footnote Frege writes: 'To judge is not merely to grasp a thought, but to acknowledge its truth.' (Frege, 1892, 34) We can assume propositions 'for the sake of the argument' and derive logical consequences from them without thereby having to take a stance on whether they are true or not. In assuming that there is a set containing all and only those sets that do not contain themselves for purposes of *reductio*, I am not asserting that proposition.

It is true that, despite his acknowledgement of the need for distinguishing assertion from assumption, Frege did not apply it as one might expect: all propositions of *Begriffsschrift* and *Grundgesetze* are marked with the judgement stroke and thus asserted. Propositions that could form assumptions in the process of reasoning appear in the antecedents of asserted conditionals.[14] It

from being embeddable or that cannot be embedded, but that are not signs of speech acts.

[14]Notice, however, how Frege proceeds in the appendix to the second volume of *Grundgesetze*: deriving Russell's contradiction in Begriffsschrift, Frege informs us that he will 'leave out the judgement stroke because truth is in doubt' (Frege, 1903, 256). Curiously, this passage is omitted by Geach and Black in their translation of the appendix. Frege here draws logical inferences from propositions that are not judged; his practice betrays his doctrine that from mere assumptions nothing can be inferred ((Frege, 1906, 387), (Frege, 1918c, 47)). There would, hence, be a way of expressing mere assumptions in Frege's logical practice, namely, by refraining from applying the judgement stroke. But this is not a method Frege uses in his official development of logic. At the beginning of *Grundgesetze* and elsewhere, a formula without a judgement stroke attached is taken to be the name of a truth value. A proposition can only ever name a truth value, be it the True or the False. To judge is to take the step from the sense of a sentence, the thought, to its reference, its truth value (Frege, 1892, 35). In judging, we proceed from a thought to its truth value, or rather from the thought to the True. According to Frege's official doctrine, inference requires that process to have been made. See Textor's reconstruction of Frege's theory of judgement, where he explains:

took later developments until systems of logic were formalised that deploy Frege's insight. According to Gentzen, the main difference between his systems of natural deduction and the 'logistic' calculi, as he called them, is that in his systems deductions begin with formulas that are assumed, rather than with axioms that are asserted:

> The essential difference between NJ-derivations [i.e. in natural deduction for intuitionist logic] and derivations in the systems of Russell, Hilbert and Heyting is the following: In the latter, correct formulas are derived from a number of 'logically basic formulas' [i.e. axioms] by means of few rules of inference; natural deduction, however, does not in general start from logically basic propositions, but from a s s u m p t i o n s [...], which are followed by logical inferences. A later inference then makes the result again independent of the assumption. (Gentzen, 1934, 184)[15]

Roughly around the same time Jaśkowski makes virtually the same observation:

> In 1926 Prof. J. Ł u k a s i e w i c z called attention to the fact that mathematicians in their proofs do not appeal to the theses of the theory of deduction, but make use of other methods of reasoning. The chief means employed in their method is that of an arbitrary supposition. The problem raised by Mr. Ł u k a s i e w i c z was to put these methods under the form of structural rules and to analyze their relation to the theory of deduction. (Jaśkowski, 1934, 5)

Jaśkowski solves Łukasiewicz's problem by formalising a system of natural deduction. Like Gentzen, Jaśkowski continues to point out that assumptions are made to be discharged, that an implication derived from a conclusion derived under a supposition does not depend on the supposition: 'It would remain true even in case the suppositions used [in its derivation] should be false.' (Jaśkowski, 1934, 6) Both Gentzen and Jaśkowski underline that making and discharging assumptions is essential to the process of logical inference as captured by natural deduction.

Making an assumption is not often listed amongst examples of speech acts. It is, however, quite clear that to make an assumption is to perform a speech act. It is to do something with the content of a sentence, with a proposition or a thought, and to engage in a linguistic activity. An assumption can have the same content as an assertion, a question, a command or a wish and is distinguished from them by what is done with the content. Dummett concurs that 'in supposition, a thought is expressed but not asserted: "Suppose ..." must be taken as a sign of the *force* [...] with which the sentence is uttered.' (Dummett, 1981, 309) Although this observation is virtually immediate once the distinction between force and content is drawn, it is possible to give more evidence and argument for it. Doing so contributes to an analysis of this speech act. I shall follow Jaśkowski and call the speech act of the making of an assumption *supposition*.[16]

'Judgement and inference are "level-crossing" mental acts. In them the judger advances from a thought to its truth-value.' (Textor, 2010, 639)

[15] This differs slightly from Szabo's translation (Gentzen, 1969, 75).

[16] For a detailed analysis of the norms governing suppositions and how supposition differs from

Supposition shares features with other speech acts. It is similar to requests and commands in that suppositions are often expressed using the imperative: 'Let a be an F', 'Assume p', 'Suppose q'. If the use of the imperative indicates a speech act, this suggests that its use in supposition does so, too.

Supposition has a specific purpose: it marks the first step in argumentation or logical deduction. Supposition comes with the intention to draw an inference. Gentzen and Jaśkowski go even further: the intention is to produce a chain of inferences with the aim of discharging the assumption made. Thus, like commands, requests and questions, suppositions prompt further actions, the former answers and the carrying out of the command or request, the latter further steps in an argument or deduction. If in the former cases, this is due to the fact that speech acts have been performed, this suggests that supposition is also a speech act.[17]

There are conventions marking assertions, questions and commands. Often these are not sure-fire indications, but in general fair enough to determine which speech act has been performed. If a sentence ends in a full stop, this is a rough and ready indication that it is an assertion; if it ends in a question mark, this is a rough and ready indication that it is a question; if it ends in an exclamation mark and is in the imperative mood, this is a rough and ready indication that it is a command, if issued by a person with the relevant authority. There are also conventions marking when something has been assumed. In the four most popular systems of natural deduction, these are quite precise rather than rough and ready:

(1) By writing formulas at the top nodes of a proof tree with no line on top (Gentzen);
(2) By writing an S at the beginning of the formula and a numeral to the left, with a prefix if the assumption is in the scope of other assumptions (Jaśkowski);
(3) By writing 'hyp' to the right of the formula and $|_{-}$ to its left, with further lines $|$ to the left if the assumption is in the scope of other assumptions (Fitch);
(4) By writing an assumption number to the left of the formula and an 'A' to its right (Lemmon).

Thus, just as there are conventions marking the speech acts assertion, question, command and request, there are conventions marking supposition. This feature, too, puts supposition into the realm of speech acts. Notice that there is no conventional mark indicating that a thought is expressed: the sentence expressing the thought suffices.

The test provided by the Frege Point provides further reasons for counting supposition amongst the speech acts. If the conventional signs of supposition are such that they cannot be embedded, this is an indication that it is a speech act. And this is indeed the case. Conventional signs for supposition in ordinary

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other speech acts, see (Green, 2000). Green also remarks, as I will below, on the fact that there are conventional ways of marking supposition in natural deduction, showing that supposition is a speech act.

[17] When I presented an early version of this material at a work in progress seminar in London, Mark Textor asked whether one can't just suppose without drawing inferences. Keith Hossack responded that this is not supposition, but entertaining a thought.

English, such as 'Suppose *A*', 'Let *a* be an *F*' and 'Assume *p*' cannot be embedded. 'If suppose *A*, then *B*', 'Suppose assume *A*', 'It is not the case that let *a* be an *F*' are gibberish. Similarly for the conventional signs of supposition in formal systems of natural deduction. Expressions such as '$(Sp) \supset q$', '(1. |₋ *p* hyp) $\supset q$' and (1 *p* A)$\supset q$ are illformed, and although the formula that occupies a top node of a proof tree can occur in the antecedent of a conditional, it makes no sense to put the top nodes of proof trees into that position. In Jaśkowski's and Fitch's system, assumptions can be made in the scope of other assumptions, but this is not the same as embedding an assumption in another. To make an assumption in the scope of another is to perform two speech acts one after the other. It is not to embed one speech act in another. No provision has been made for strings of symbols such as '1. 2. *SSpq*' or '|₋ |₋ *p* hyp *q* hyp', or similar strings with *q* omitted. But as the systems of Gentzen and Lemmon show, the notion of the scope of an assumption is not essential. Be that as it may, no provision has been made in Lemmon's system for strings of symbols such as '1 2 *p* A *q* A' either, and in Gentzen's system, where supposition is effected by writing a formula on top of a line indicating an inference with nothing above it, there isn't even anything that might count as an attempt to embed one supposition within another.

Finally, although we can describe or report that an assumption has been made by a sentence such as 'It is assumed that *p*', this is not the same as supposition. It does not have the same effect. We cannot render what is being done when an assumption is made by the phrase 'It is assumed that'. The inference 'It is assumed that *p*, it is assumed that $p \to q$, therefore *q*' is invalid: It may be true that it is assumed that *p* and that it is assumed that $p \to q$, while it is false that *q*, because it is possible to assume falsehoods. Nonetheless, from the supposition that $p \to q$ and the supposition that *p*, *q* follows logically. 'It is assumed that' is a sentential operator, not an indicator of a speech act: it can be used to describe or report which assumptions have been made, but 'It is assumed that *p*' cannot take the place of performing the speech act of assuming that *p*. The description or report may be true or false; the assumption is made or not. Compare with Frege's assertion sign: it indicates the assertoric force of an inscription without asserting that the inscription is asserted; the latter is done by means of the sentential operator 'It is asserted that'. Like assertion, supposition is not something that is part of the proposition assumed. It is something that is done with a proposition.[18]

Supposition is different from merely grasping or expressing a thought. Thoughts can be grasped without being assumed, e.g. when I grasp the components *p* and *q* in complex sentences such as $\neg p$ and $p \to q$. We can test whether someone has grasped a thought expressed by '*p*' by asking 'Do you understand this sentence?'. Even when the answer is 'Yes', this need not be the preparation for a chain of reasoning. Grasping or expressing a thought need not be followed by inferences.

As pointed out by Frege, supposition is evidently something other than assertion. For further illustration, consider Descartes at the end of his first

[18]As Frege says, to judge is something utterly peculiar and incomparable. (Frege, 1892, 35) Nothing other than a judgement has the effects of a judgement; in particular, a description that a judgement that *p* has been made (by someone or other) need not involve a judgement that *p*. Van der Schaar gives an account of the difference between judging and describing a judgement, of the first person perspective and the third person perspective on judgements (van der Schaar, 2018). Similar remarks apply to supposition.

meditation: 'I will suppose [...] that there is an evil spirit who is supremely powerful and intelligent, and does his utmost to deceive me.' (Descartes, 1954, 65) Descartes assumes this, but does not assert it: he assumes it to see what follows in his quest for a rational reconstruction of his beliefs on firm foundations. It is also an assumption to be discharged. Descartes aims to draw conclusions that do not depend on this assumption. Anything can be assumed, at least in formal logic, and maybe even in philosophy, but assertion is governed by stricter norms and not anything can be (felicitously, sincerely) asserted. Not many people have ever been in a position where they would assert 'I am being deceived by an evil spirit'.[19] The assertion that p does, the supposition that p does not, commit to the truth of p. Speakers use assertions to express their beliefs; they do not use suppositions for that purpose. Although supposition has features in common with command, question or request, it is a speech act that differs from them, too. As Dummett observes, a command can be followed up by a question 'Have you done it yet?', but 'Let a be an F' or 'Suppose p', can't be followed up by such a question. (Dummett, 1981, 309) For a similar reason, suppositions are not requests either. Supposition is also different from asking a question: I can assume a proposition without wondering whether it is true or not. A question is a challenge to provide an answer; a supposition can be made without any view on settling the question whether it is true or not – indeed, once an assumption is discharged, its truth value is irrelevant to the truth of the conclusion.

5 Supposition as a Problem for Bilateralism

Formulas of bilateral logic are prefixed by + or −, representing the speech acts of assertion and denial. Being a speech act, supposition requires a propositional content as that which is supposed. An assertion or a denial is not a propositional content. Thus it is not possible to assume an expression such as $+A$ or $-A$. Every formula of bilateral logic is already put forward with assertive or rejective force. There is therefore no sense to assuming such a formula. Speech acts cannot be embedded. 'Assume $+A$' and 'Assume $-A$' are therefore meaningless.

Nonetheless, formulas of the form $+A$ and $-A$ are supposed to feature as assumptions in deductions in bilateral logic. The rules $+\supset I$ and Reductio show as much. They permit the discharge of signed formulas. The conclusion of an application of these rules no longer depends on the signed formulas discharged. That which is discharged is an assumption.

As pointed out by Jaśkowski and Gentzen, supposition is an essential feature of inference. But we cannot make sense of the notion of making an assumption in bilateral logic, where every formula is prefixed with a sign for assertion or denial. Bilateral logic demands we do something that cannot be done: to embed the speech acts of assertion and denial within the speech act of supposition. Bilateral logic as it stands is thus incoherent.

[19]Saints Ignatius of Loyola and Teresa of Avila came close. Both report in their autobiographies the realisation that some of the thoughts and feelings that arose during their meditations were temptations and effectively assert that they were being deceived by evil demons. But even they do not quite report having asserted 'I am being deceived by an evil demon' in the present tense. Note the difference: For Descartes, the thought 'I am being deceived by an evil demon' occurs within disinterested philosophical reflection. For Teresa and Ignatius, it is the cause of extreme distress. The difference between the supposition and the assertion couldn't be more dramatic.

According to bilateralists, + A and − A can be rendered as propositional questions and their answers. Doing so starkly presents the predicament. 'Suppose was the accused in Berlin at the time of the murder? Yes' makes no sense. 'Suppose is it the case that A? No' and 'Assume is it the case that A? Yes' are of the same kind of gibberish as Rumfitt's example to illustrate that + and − cannot be embedded (p.6), and so is 'Let is a an F? Yes'.[20]

We can assume that something has been asserted or that something is assertible. 'Suppose it is asserted that A' or 'Assume that A is assertible' make sense. But they are different from assuming that A. To assume a proposition is not to assume that anyone asserted it. To assume that A is assertible is different from assuming that A. If B follows from the assumption that A, then I can infer 'If A, then B'. If B follows from the assumption that A is assertible, I can infer that 'If it is assertible that A, then B'. These are not the same. To take an example of Dummett's, let A be 'You will go into that room' and B 'You will die before nightfall', so that in 'If you go into that room, you will die before nightfall', 'the event stated in the consequent is predicted on condition of the truth of the antecedent (construed as in the future tense proper [i.e., not the future tense expressing present tendencies]), not of its justifiability.' (Dummett, 1993, 193) Suppose that the present tendencies are that you will go into that room, but you later change your mind, don't go and don't die before nightfall. Then the conditional 'If it is assertible that you go into that room, you will die before nightfall' is false, as the antecedent is true and the consequent is false, while the conditional 'If you go into that room, you will die before nightfall' is true, if the room is one in which everyone is killed who enters before nightfall. The distinction between assuming that a proposition is assertible and assuming the proposition is pertinent for bilateralists like Price and Rumfitt, for whom a crucial aspect of the motivation for adopting the bilateral approach to meaning is their claim that it enables them to draw the distinction between truth and assertibility. (See (Price, 1983, 167) and (Rumfitt, 2002).) Besides, 'Suppose it is asserted that A' or 'Assume that it is assertible that A' cannot render correctly the bilateralists' attempts at assuming + A and − A: + A and − A represent speech acts of assertion and denial, not reports that any such speech acts have been performed or assertions that they could be performed.[21] 'It is asserted that' and 'It is assertible that' are sentential operators, not indicators of speech acts.[22]

[20]How about 'P? Suppose Yes' or 'Let a be an F? Yes'? See section 6.

[21]To assume that an assertion has been made or a question answered is irrelevant to logic, or at least it does not cover all the cases logic is concerned with: Descartes need not have asserted that he is deceived by an evil demon, nonetheless he and we can proceed from that assumption and see what follows, draw consequences and potentially reject the assumption, if we reach a contradiction. And rejecting an assumption here means: to derive its negation, which we are then entitled to assert, if we assert also all the other premises used in the argument. Rejecting an assumption in the sense relevant to logic is not like rejecting an assertion (as in metalinguistic negation): it is the step after deriving a contradiction (or otherwise unpalatable proposition) from it (and other assumptions or asserted propositions), that is, it is to derive and assert its negation (on the basis of other assumptions).

[22]The items to which bilateralist logic is applied can hardly be possible assertions. I doubt that mainstream bilateralists are happy to admit that there are possible assertions, so the only way to make sense of the claim that A is a possible assertion is to say that A is assertible. Maybe bilateralists could reject the Frege Point and adopt a view that aims to imbue propositions with an intrinsic assertoric or rejective force, a force that is canceled if they are embedded into other speech acts, such as supposition? Jesperson argues forcefully against such a view (Jespersen, 2021). The view also goes against the evidence provided by Rumfitt that bilateralists accept the Frege Point and do not think speech acts can be embedded.

Maybe bilateralists could respond that their logic is one that works without supposition: the rules specify how to proceed to further assertions and denials from assertions and denials that have in fact been made.[23] Compare with Frege's view that only from true premises can something be concluded. We can, however, discount this option: the framework of natural deduction chosen by bilateral logicians betrays that this cannot be the intention, as it does not fit the Fregean account of inference. Seeking a way out along the Fregean route and providing an axiomatic system of logic in which some axioms are asserted, others denied is also not conducive to the expressed aim of providing an inferential semantics for the logical constants: it is to give up on the project of specifying the meanings of the connectives in terms of rules of inference.

The notion of the discharge of assumption merits further consideration. It is the second essential aspect of inference pointed out by Gentzen and Jaśkowski. In unilateral systems of logic, if a rule permitting discharge of assumptions is applied, the conclusion no longer depends on their truth. What could it mean to discharge a speech act of assertion or denial? Discharging an assumption is not like retracting an assertion. To see this, it suffices to compare Frege's retraction of Basic Law V and Descartes's discharge of the assumption that he is being deceived by an evil spirit. But a few more words may be in order. If an assertion has been made, or a question raised and answered, a speech act has been performed. And even though I can retract an assertion or change my mind what the answer to a question is, the assertion or question and answer cannot be made undone: they are events that have happened, and we cannot, as it were, remove them from the universe by a process such as applying implication introduction or *reductio ad absurdum*. It is possible to cancel the commitment to the correctness of an assertion previously made, but that is not like discharging an assumption: cancelling a previous assertion is not done by an application of a rule of inference. Cancelling a commitment to a previous assertion is not analogous to the process of inferring further propositions that have the content of the assertion as a component; indeed, no such process would appear to make sense, as it would appear to require having the assertion as a component. Discharging an assumption is also a notion bilateralists cannot make sense of. There is no process that does to the speech act of assertion (or denial, for that matter) what discharge does to assumptions.

That it makes no sense to discharge an assertion is further evidence that we cannot assume assertions either: the possibility of its discharge is an essential feature of making an assumption.

What about Smiley's and Rumfitt's 'readily comprehensible variety of actual deductive practice' that uses propositional questions and their answers? There is no need to appeal to the bilateralist machinery to make sense of such inferences. One may, instead, appeal to Textor's account, who argues that 'Yes' and 'No' used in answering questions are not force indicators, but prosentences (Textor, 2011). Thus any such aspect of deductive practice can be reconstructed without appeal to speech acts of assertion and denial.

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[23] As Dorothy Edgington and Mark Textor wondered.

6 Attempts to Solve the Problem

One might object that the difficulty pointed out in the last section is less than a problem and more of an omission: bilateral logic is incomplete and needs to recognise further speech acts besides those marked by + and −; in particular, it needs to recognise also the speech act of supposition.[24] Could we not answer 'Suppose yes' to questions such as 'Was the accused in Berlin?' and mark the speech act of supposition thereby? The first thing to note here is that this is not what Smiley, Rumfitt and Humberstone are doing, according to whom + and − are to be read as assertion and denial, not supposition.[25] 'Yes' and 'No' are not 'Suppose yes' and 'Suppose no', and if the former are represented by + and −, the latter are not represented by them, and hence we should have to add further signs for supposition.

Such an approach has been followed up by Kearns (1997). Kearns is a bilateralist at heart: he accepts that there are primitive speech acts of assertion and rejection, which he represents by $\vdash$ and $\dashv$. Correspondingly, there are two kinds of supposition, supposing as true, which he represents by $\llcorner$, and supposing as false, represented by $\neg$ (Kearns, 1997, 335). Every formula in a deduction is signed by one of these four symbols. To keep the system simple, Kearns only considers rules for $\vdash$ and $\llcorner$, and he only explicitly states some of the rules for conjunction, disjunction and negation. Even so, the system brings with it certain complications, as it needs to be settled what to do with conclusions that are derived from a mixture of asserted and supposed premises. This leads to a large number of rules: conjunction introduction, for instance, has three forms. Kearns has a principle for deciding whether the conclusion of an inference is asserted or supposed: if the only suppositions on which the premises of a rule depend are those to be discharged by its application, then the conclusion is asserted; otherwise it is supposed. Only suppositions can be discharged.[26]

There is, however, no need to consider all these variations. A rule of inference of Kearns's logic is that from an assertion of A, its supposition follows: from $\vdash A$ infer $\llcorner A$. It would therefore suffice to formulate only rules for when all premises are supposed and leave the cases with asserted premises as derived rules of inference. What is more, as a consequence of Kearns's principle for deciding whether a conclusion is asserted or supposed, it suffices to give rules that conclude with suppositions, and then add the global condition that the

[24] Mark Textor suggested I put my point like this. The following also owes to discussions with Greg Restall.

[25] Evidently, we can't read + and − as 'Suppose the accused as in Berlin etc.? Yes' as that, if we admit it at all, asserts that it is supposed that the accused was in Berlin, and this is different from supposing that the accused was in Berlin. Cf. pp.10 and 12.

[26] There are unexplained question marks in the rules for disjunction and negation elimination (Kearns, 1997, 336). I interpret them as meaning that these formulas may either be asserted or supposed. There is a typo in negation elimination, a version of classical *reductio ad absurdum*: negations are missing from the discharged suppositions. Vacuous discharge appears to be forbidden in disjunction elimination, which is why its minor premises can only be supposed, while in negation elimination, it is permitted above supposed premises. Kearns says versions of *ex contradictione quodlibet* (vacuous discharge above both premises in negation elimination) are valid, as long as at least one premise and the conclusion are supposed; if both premises are asserted, it is not permitted to proceed to the assertion (and presumably the supposition) of an arbitrary formula; rather, 'once a person finds herself [in such a position], she must abandon some of her beliefs' (Kearns, 1997, 337). Abandoning a belief is then not discharging it and deriving the assertion (or supposition) of its negation.

conclusion of a deduction is asserted if all the premises it depends on are asserted, supposed otherwise. It is clear, however, that prefixing $\llcorner$ to all premises and conclusions is superfluous. The situation is thus exactly as in a system such as Gentzen's, where assumptions are not marked in any special way, and a deduction of A from formulas Γ entitles us to assert A if we assert all formulas in Γ.[27]

The fact that Kearns permits suppositions to be conclusions of inferences presents a more general problem. Kearns's notion of supposition is prised apart from the notion of discharge. For instance, if $\llcorner A \wedge B$ is concluded from $\llcorner A$ and $\vdash B$, it cannot be discharged further down in the proof. Kearns considers supposition to be something weaker than assertion. In some sense or other that may be true: a supposition does not commit in the way an assertion does. But no such sense is pertinent for logic. Supposition is not a weaker kind of assertion, but something different altogether. Assumptions stand at the beginning of deductions and are not the result of inference. And as Gentzen and Jaśkowski observe, assumptions are made to be discharged: it is of the essence of an assumption that it may be discharged by an application of a rule of inference further down in the deduction. Consequently, in their systems of natural deduction, assumptions *only* stand at the beginning of deductions and introduce the formulas from which the deduction is going to take its course. I conclude that Kearns's $\llcorner$ does not represent the speech act of supposition, as supposition plays a different role from that played by $\llcorner$ in Kearns's system.[28]

The forgoing considerations also shows Kearns's rule 'from $\vdash A$ infer $\llcorner A$' to be absurd. A conclusion drawn on the basis of a deduction in which all assumptions are discharged is asserted outright and there is no sense in which it is supposed. Kearns, however, must say that it is, as according to him, from the assertion of the conclusion of this deduction, its assumption follows.

These problems are not just problems that mar Kearns's approach. Any approach that insists on adding speech acts of supposition to those of assertion and denial would need to answer the questions Kearns has aimed to address, and if all formulas in a deduction are supposed to be marked by signs for a speech act, the question remains what speech act is supposed to follow from supposed formulas.

Maybe the most reasonable thing would be not to sign conclusions of inferences by markers for speech acts at all. Evidently, even marking conclusions

[27] In Jaśkowski's system, suppositions are marked by S, while asserted formulas are not marked by anything. 'The above conventions [of how to construct deductions in his system of natural deduction] lead us to some new expressions [those beginning with 'S'] which must be considered as significant ones. [...] We shall retain for the term "proposition" the meaning already given, namely the significant propositions of the usual theory of deduction' (Jaśkowski, 1934, 7), i.e. an axiomatisation of the propositional calculus by Łukasiewicz in which all formulas are asserted. There would be no need for a separate symbol indicating supposition, as suppositions are already marked by their position in the deduction, standing, as they do, to the right of prefixes composed of numerals indicating scope, each supposition with its unique prefix. Propositions, whether concluded by the discharge of suppositions or used as premises, do not get prefixes. It is worth noting that formulas concluded under suppositions are not marked by anything other than the prefix of the suppositions under which they stand.

[28] The act of drawing a conclusion is often marked by 'therefore': 'Suppose A and suppose B, therefore suppose $A \wedge B$' is once more gibberish, to use Rumfitt's word, and hence, as according to Kearns concluding $\llcorner A$ is meaningful, $\llcorner$ cannot be a sign for the speech act of supposition that is pertinent to logic. One will observe that putting a question and answer after 'therefore' does not fare much better.

only by + and − if they are concluded exclusively from asserted and denied formulas would open up a problem of Kearns's approach again: what should we conclude if one premise of a rule is signed and the other isn't. And so we are back to Jaśkowski.

One might try to solve the problem posed by supposition for bilateralism by observing that even if speech acts cannot be embedded, there are speech acts that are conditional: there are conditional commands, requests and bets, for instance, such as 'If you go to the shop, get some beers' or 'I bet a tenner it'll rain if I don't take an umbrella'. The request and the bet are made on condition of other things taking place. If you don't go to the shop, the request is void. If I take an umbrella, the bet is off. It is plausible to add conditional assertion to the list of conditional speech acts. Indeed, it is plausible that if a conclusion is drawn on from assumptions, it is asserted conditionally on those assumptions. An expression like 'therefore' that signals an inference also bears the marks of a speech act. Consider this example: Let a be an F. But no F is a G. Therefore, a is not a G. Here we have three speech acts: a supposition, an assertion and the announcement of a conclusion. The result is a conditional assertion: The conclusion that a is not a G is asserted conditionally upon a's being an F. There is also a practice of recording the conditional assertion resultant upon a deduction as $\Gamma \vdash A$ and of calculi for deriving the further commitments incurred by conditional assertions, i.e. single conclusion sequent calculus. A deduction, then, consists in a series of speech acts: it begins with suppositions or propositions the truths of which are accepted, continues with announcements of propositions inferred, and results in a conditional assertion of its conclusion, if any assumptions remain undischarged, or its outright assertion, if not.[29]

This is an attractive way of understanding the result of a unilateral deduction. The question is how to apply it to bilateral logic.

If there are conditional assertions, the bilateralist can add conditional denials. The product of a deduction is a conditional assertion or a conditional denial. Undischarged formulas $+ A$ and $- A$ should then represent the conditions on which the conclusion of the deduction is asserted or denied. But this does not get the conditions right. A speech act conditional upon the assertion or denial of a proposition is different from a speech act conditional upon its truth or falsity. A conditional request, command or bet is conditional upon the truth of the proposition expressing the condition, not the performance of a speech act with that proposition as its content. Similarly, the condition of a conditional assertion should be expressed by A or $\neg A$, as it is done in a unilateral system, not by $+ A$ or $- A$, as the bilateralist would have to insist. It is crucial to the bilateralist that + and − are speech acts: the bilateralist needs to ensure that + and − are not merely notational variants of the trivial truth function and negation, and the way to do this is to insist on their status as speech acts. A must be different from $+ A$, $\neg A$ from $- A$. But if $+ A$ and $- A$ mark the conditions of the speech act, they are no different from A and $\neg A$.

This is also seen by looking at the conditional. A conditional assertion of the kind that is at issue here, one put forward on the grounds of a deduction of

[29] Eliot Michaelson, Michael Potter and Bernhard Weiss pressed me on this issue, and it is to them that I owe the objection. In our four systems of natural deduction, it is not marked whether any of the formulas from which the deduction proceeds are accepted as true unless they are axioms: they are all treated as assumptions. But it would be straightforward to add a convention for indicating such formulas, the most immediate one being to treat them the way axioms are already treated.

a conclusion from assumptions, is equivalent to the assertion of a conditional. This follows from the bilateral rules for ⊃.[30] This shows that the condition of the assertion is A, not $+A$: $+A$ cannot go into the antecedent of the conditional. Only A can go there.

7 Conclusion

The core of the argument of this paper is the following. Supposition is a speech act. In bilateral logic, the premises and conclusions of inferences are asserted or denied. Speech acts cannot be iterated. Thus there cannot be any assumptions in bilateral logic. But this is absurd: assumptions and their discharge are essential to logic.

One might protest: something in this argument must be wrong, for if that were so, then what is it that bilateral logicians are doing when they assume and discharge formulas of the form $+A$ and $-A$. My best diagnosis is that the practice of bilateral logicians shows that their + and − are non-embeddable truth and negation operators. The description of − and + as speech acts does not match their use.

All this may just point to an incompleteness in the bilateral account of deduction: other speech acts need to be acknowledged besides those marked by + and −. However, an attempt of doing just that has been shown to be inadequate. It is also not adequate to read deductions in bilateral logic, analogously to a plausible way of reading deductions in unilateral logic, as conditional assertions and denials. The burden of proof lies on the bilateralist to indicate how the account is to be amended.

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[30]This marks a difference between conditional assertion and conditional requests and commands. There is a difference between a conditional bet and a bet on a conditional; a conditional request and a request of a conditional. If bet on the conditional, that if I don't take an umbrella, then it will rain, I've won if I take an umbrella. If I request the conditional that if you go to the shop, then you buy beer, you've complied if you don't go to the shop. By contrast, there seems to be no such distinction in the case of assertion: a conditional assertion and the assertion of a conditional amount to the same thing, at least in the system we are considering, where the deduction theorem holds. If the condition of a conditional assertion reached by deductive inference is not fulfilled, although I am not committed to the assertion of the conclusion, I am still committed to the conditional that follows by implication introduction. In conditional assertion, I am committed to asserting the conclusion if the condition holds; in the assertion of a conditional, I'm committed to the consequence drawn by *modus ponens*, if the condition holds. In this respect conditional assertion is thus different from conditional bets and requests.

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